AN OUTSTANDING NONSENSE IN QUANTUM MECHANICS

The use as a physical theory of the quantum fields theory in the standard Lamb shift calculation

Speech for the abandon of all language

using the complex numbers in quantum mechanics

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Abstract. The Lamb shift calculation achieved during the years 1948-1949 is a great work, certainly the most admirable after the publications in 1928 (Darwin, Gordon, Tetrode,..) on the Dirac theory of the electron. It is presented as an "outstanding triumph" of the so called Quantum Fields Theory. But though the calculation is mathematically irreproachable, it contains a nonsense, unsurpassable in the story of sciences: the introduction of the Planck constant in the number \( \pi \).

The origin of this nonsense is based on the fact that the \( i \) which lies, multiplied by the Planck constant, in the Dirac equation is a bivector of the spacetime (Hestenes, 1967) and has nothing to do in the electromagnetic potentials as they are presented in the so-called "Theory" of the Quantum Fields.

1. The Quantum Fields Theory

The quantum fields theory (QFT) was built on ground of the works of Dirac (1927), Jordan and Pauli (1928), Heisenberg and Pauli (1929). It seems that its purpose is to express as closely as possible the photon as a particle.

For describing the two ways of its construction we follow the treatise of W. Heitler [1], considered as the usual reference for the statement of the QFT:

1. A mathematical construction. One considers a "vector potential \( \mathbf{A} \) which, according to Par. 6 eq. (14) may be written as a series of plane waves" ([1] Par. 7, p. 56, lines 1-3)

\[
\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_0^* \tag{1}
\]
where $A_0$ is complex and $A_0^\ast$ is the complex conjugate of $A_0$ ([1], Par. 6, eq. (14)). In such a way that $A$ is real in agreement with "the classical radiation theory" ([1], Par. 6) and so the laws of the classical electromagnetism are strictly respected.

**Question:** Is it possible to consider the potential $e/r$ as a series of plane waves?

2. **A rule presented as physical.** The imaginary number $i = \sqrt{-1}$ is replaced, in the application of the previous construction to some operators, by $i\hbar$ where $\hbar$ is the reduced Plank constant. This replacement is justified

"by exact analogy with the ordinary quantum theory"

([1], Par. 7, p. 56, lines 10-11).

The origin of the above rule is in the fact that $\hbar$ appears in the electromagnetic fields in quantum theory, and that, in the quantum theory of the electron, the number $i$ appears in the association $i\hbar$, with $\hbar$.

Though the replacement is placed in operators ([1], Par. 7, eq. (6)), as a necessity, it is translated in the expression of the potentials given by the equation (1). This equation is not affected by such a replacement, because if you multiply $i$ by whatever you want, this equation remains valid, since the imaginary part of its right hand part cancels. Nevertheless because of the presence of $\hbar$ in the above expression of the potential, the field is considered as "quantified".

**It is exactly in this way, association $i\hbar$ of $\hbar$ with $i$, that the potentials appear in the standard Lamb shift calculation.**

So the QFT is, mathematically, in a strict respect with the laws of the electromagnetism.

One can be stopped in the opinion that the association of $i$ and $\hbar$ brings something new, since the imaginary of the right hand part of (1), and so $\hbar$, cancel. But the fact that this association appears in operators and not directly in (1) makes the event less evident.

This construction can bring an real interest in some cases, in particular when several photons are to be considered in a same part of the space. On my own part, each time I have met the QFT in the theory of hydrogen-like atoms, I found that it brings rather serious complications
with respect to the simple application of the laws of the electromagnetism in its strictly real form.
Furthermore the QFT seems completely absent from the electroweak and chromodynamics theories (see [2]).

2. The Quantum Theory of the Number $\pi$

Despite its mathematical validity, the QFT was critisized by several authors as defining a physical theory, in particular A.O. Barut, who have supported that the reason of the presence of $\hbar$ in a field is to be associated not with the laws of the electromagnetism, but with the presence of $\hbar$ in the source of the field. Such a point of view is called "semi classical" but is not considered, presently, as a physical theory by the quasi totality of the physicists. (However Barut told me, on ground of a conversation that he had with Dirac, that Dirac himself was, at his end of his life, no longer convinced of the validity of the QFT).

Suppose you are, not at all a physicist, but rather a geometer, and that you have found in the fundamental article [3] of D. Hestenes that the $i$ of the Dirac equation is in reality a bivector $e_2 \wedge e_1$ (or $e_1 \wedge e_2$, following the two possible orientations of the spin) of the Minkowski space-time $M = R^{1,3}$, so a real object equal to $e_2 e_1$ in the real Clifford algebra associated with this space and whose square in this algebra is equal to -1 (Note that $i$ had already been replaced by Sommerfeld [4] and Lochak [5], in an expression of the Dirac spinor, by $\gamma_2 \gamma_1$, whose square is equal to -1 (I unit 4 × 4 matrix) in which the Dirac matrices $\gamma_\mu$ were implicitely identified to the vectors $e_\mu$ of a galilean frame).

The product by $\hbar c/2$ of this bivector, after its transformation by the Lorentz rotation which changes the direction of the time axis of the laboratory frame into the one, considered at each point $x$ of $M$, of the Dirac current $j$, defines the proper angular momentum, or spin, of the electron.

One sees an indisputable source of $\hbar$ in quantum mechanics: its presence in the spin of the electron leading to the presence of $\hbar$ in the energy of the electron. In fact, the energy $E$ of the electron in a galilean frame is nothing else but the component on the time axis of the product by $\hbar c/2$ by a vector (whose dimension is the inverse of a lenght) expressing the infinitesimal rotation upon itself of the plane $P(x)$, the "spin plane", orthogonal to $j$, defined by the spin bivector (see [6], eq. (3), and particularly for the hydrogen atom, [7], eq. (13)).
As soon the QFT seems to you suspicious : the i which is used is really imaginary (if I can say !) and the i of Dirac, a spacetime bivector, is really real. In any way a bivector has nothing to do in the electromagnetic potentials.

The ”outstanding triumph” of the QFT is presented (as far as in the French dictionaries !) as allowing the calculation of the little shift of all level of energy of a hydrogen-like atom, called the Lamb shift. In fact if one searches for a confirmation of this theory, it is not in the Darwin solutions of the Dirac equation for the hydrogenic atoms, in which the central potential Ze/r is not quantified. But, also, if there is a defect in this theory, it is to be looked for in the Lamb shift calculation.

So you begin to search how \( \hbar \) is introduced in the standard Lamb shift calculation for the hydrogen atom. The task is not easy because, as a detestable simplification used by the physicists, one writes generally \( c = \hbar = 1 \). But some day you find in [1] a recall of the calculation (mainly based on [8], also [9]) in which these constants are explicitly mentioned.

The Quantum Theory of the Number \( \pi \) !

And you find ([1], Par 34, eq. (4')) a formula which allows the calculation of one of the three terms, the Electrodynamics static term \( W_S \), which compose the shift

\[
\frac{\langle \Psi_0^*(r)e^{i(k.r)/\hbar c}\Psi_n(r) \rangle \langle \Psi_n^*(r')e^{-i(k.r')/\hbar c}\Psi_0(r') \rangle}{|r-r'|} = \frac{1}{2\pi^2\hbar c} \int \frac{[(\Psi_0^*(r)e^{i(k.r)/\hbar c}\Psi_n(r))(\Psi_n^*(r')e^{-i(k.r')/\hbar c}\Psi_0(r'))]dq}{k^2} \tag{2}
\]

This formula implies a static potential \( e/|R| \), where here \( R = r - r' \) corresponds to the positions of the electron in a state of energy \( E_n \) and the same electron in a state of energy \( E_0 \).

But you observe that \( \hbar \) is not in the left hand part of the formula but is present in the right one.

And you find that one has used the following equality
\[
\frac{1}{|\mathbf{r} - \mathbf{r'}|} = \frac{1}{2\pi^2\hbar c} \int e^{i(\mathbf{k}.(\mathbf{r}-\mathbf{r'}))/\hbar c} \frac{dk}{k^2}
\]

where \(dk/k^2 = \sin \theta d\theta d\varphi dk = d\Omega dk\) in spherical coordinates, in quite agreement with the construction of the QFT: decomposition of the potential in plane waves (?) by the use of the so called "Fourier transform" of \(1/R\) (see [9], eq. (16)), apparent complex form of the potential, and with the transform \(i/\hbar = -1/\imath \hbar\) (allowed by the fact that the imaginary part of (3) cancels), association \(i\hbar\) of \(i\) with \(\hbar\).

The presence of a factor \(1/\hbar c\) in eq. (2) is due to the fact that \(k\) has the dimension of an energy because in \(\exp\left[\imath((\mathbf{k}/\hbar c).(\mathbf{r}-\mathbf{r'}))\right]\) the vector \(\mathbf{k}/\hbar c\) must have the dimension the inverse of a length, in such a way that, after the integration, the dimension of the right part of eq. (2) is the inverse of a length as its left part.

You consider (3) during some minutes, and (after at least half an hour of uncontrollable laughter) you can say (see [10], p.444):

"What has been done in reality ? One has written

\[
\frac{1}{R} = \left(\frac{\pi}{2}\right) \times \frac{\pi}{2} \text{ exact !}
\]

The Planck constant is here ! I repeat,

THE PLANCK CONSTANT IS HERE, INSIDE \(\pi/2\) !

(not inside \(2/\pi\) !)

Indeed one has

\[
\frac{\pi}{2} = \int_0^\infty \frac{\sin x}{x} dx = \int_0^\infty \frac{\sin(k'R)}{k'}dk'
\]

denoting \(k' = k/\hbar c\)

\[
\frac{\pi}{2} = \int_0^\infty \frac{\sin(k'R/\hbar c)}{k'/\hbar c} \frac{dk'}{\hbar c}
\]

and using

\[
\frac{\sin(k'R)}{k'R} = \frac{1}{2} \int_0^\pi \exp(i k'R \cos \theta) \sin \theta d\theta = \frac{1}{4\pi} \int \exp(i(k'.R)) d\Omega
\]

one deduces, after multiplication by \(2/\pi\), the equation (3)."
"You can put, it’s no so bad,
The Planck constant into your bed.
But if you don’t want I say haïe!
Please, don’t put it in the number $\pi$"

You think that perhaps this nonsense is an invention of Heitler and you
study into detail the original articles. And you find the formula (3)
in [9], eq. (16), without the mention of $\hbar c$, with however $\hbar = c = 1$
in p. 390, and, at last but not least, the fact that $k$ is indeed to be
considered as owning the dimension of an energy, in terms in the form
$k - E$ appearing in the eq. (27) giving the Electrodynamic energy term
$W_D$.
Note that the calculation of $W_D$ does not need the use of the QTF (see
[11], Addendum).
So, not only the use of QFT leads to a nonsense but it is the source of
mathematical complications.

3. Some mysteries

You are placed in front of some mysteries.

1. First on one side the potential in the form $e/r$ used in the Darwin
solution is not considered as quantified. But a potential $e/R$ of the same
form is considered as being able to be quantified. How the question of
the difference on the treating of two inverses of lenght by the laws of
Nature has been eluded?

2. How is it possible that the extravagant transformation of $1/R$ has
been placed in the works of physicists among the most famous of the
last century? Is there a volontary trickery? Surely not! An expla-
nation: their confidence in the QFT was so strong, also their intuition
for finding the result was so genius, that they were not very attentive
about the details.

3. How is it possible that during forty years the nonsense has been seen
by nobody? Again the confidence in the QFT.
4. Now that the nonsense has been put in evidence and published ([10]) how is it possible that it continue to be ignored by the quasi totality of the physicists?

You try to indicate the nonsense to the physicists you meet. But, either they use the same real language in quantum mechanics as you, and you obtain laughters, either they are standard physicists, and you suffer grimaces. And what can you say? Some of these last physicists are authors of wonderfull experimental works when, in this domain, you are just able to replace a bulb of electric light.

You write to a very distinguished physicist, well acquainted with the QFT (as he says), a letter in which you mention the formula (3), adding by irony that you have discovered the ”quantization of the number π”. He makes exactly the same calculation as you and, thinking that is you who is the author of the nonsense, he answers that you have written a stupidity (as you are polite, you prefer the word nonsense).

I mention all these details to emphasize the difficulties I have in the communication of an information which contradicts a theory adopted by the totality of the physicists eighty years ago.

Per viginty annos clamavi in deserto!

4. The risks of the use of the complex languages in quantum mechanics

1. In the quantum fields theory.

I have proved in [10] that the standard Lamb shift calculation is mathematically irreproachable, but also if the use of the quantum fields theory in this calculation is considered as an ”outstanding triumph” of this theory, this calculation is an outstanding triumph of the non validity of this theory as a physical theory.

However, as long as the classical laws of the electromagnetism will be valid, there is no risk in the use of the QFT, at least as it has been built in the above presentation, except the risk of the employ of unacceptable devices.

Perhaps the use of the QFT brings mathematical commodities in cases more complicated as the simple theory of an hydrogen-like atom con-
sidered as subjected to one exterior potential, and furthermore this use may light the understanding of some phenomenas.
I suppose that the theoretical studies leaving to the admirable experimentations on the slowing down of the atoms to their quasi motionlessness have been achieved with the help of the QFT.
Perhaps new experiments have modified the QFT in such a way that it becomes a true physical theory of the photon.

*I any way, I have shown that the use of a complex formalism may lead to a nonsense.*

*The question may be posed: Does there exist, with the use of complex formalisms, nonsenses or faults in other parts in quantum mechanics?*

2. In the complex matrices and spinors formalism.

Indeed the complex language of the Pauli and Dirac matrices and spinors may lead to anomalies (see [12], sect. 4).

Furthermore it masks important geometrical properties. An example. As established quite independently by several authors (see [5], [13], [3], [14]) the gauge $U(1)$ in the theory of the electron is the group of the rotations upon itself of the ”spin plane” $P(x)$. But if you tell that to a standard physicist, he will say that the Dirac spinor $\Psi$ contains [by means of the action of the $\gamma_{\mu}$] a representation of the Lorentz group and since $i\Psi = \Psi i$, $U(1)$ is to be considered as a direct product with this representation and not as a sub-group of $SO(1,3)$ and that so $U(1)$ corresponds to an internal abstract property.

*What does that mean an ”internal property” which cannot be placed in the geometry of space-time?*

A good verification of the validity of a quantal theory may be made by its translation in the real formalism based on the use of the real Clifford algebra of $M = R^{1,3}$, introduced in the theory of the electron by David Hestenes [3], in which the Dirac spinor is replaced by a real object, a biquaternion which contains a Lorentz rotation (about in the same way that a quaternion contains a rotation in $R^{3,0}$), which does not need, for its use, the employ of matrices.
The only operator which is to be considered, in his formalism which uses only geometrical objects, is the gradient operator of $M$. 

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That leads to a considerable simplification of the language, because, in particular, it is directly placed in the geometry of the space-time which is the frame of the observations in experiments.

Also, to the extend that the Dirac spinor is implied in the theories of quarks and leptons, this formalism covers all the present theories in quantum mechanics.

I have achieved in [15] a faithful traduction of the G.S.W electroweak in which the $U(1)$ and $SU(2)$ gauges are translated into subgroups of $SO(1,3)$ acting on sub-frames of moving frames of $M$, and in which energy-momentum tensors, whose traces appear in the lagrangian, may be translated into the product of suitable physical constants by the infinitesimal rotations of these sub-frames into themselves. I have found no defect in the standard complex language for this theory, except its intricacy and that it masks this important geometrical link between the gauges and what we call energy.

In contrast, I have not found a traduction in the real formalism of the $SU(3)$ gauge of the chromodynamics theory. However the traduction of this theory can be made possible, without changing the value of the lagrangian of the theory (see [12], sect. 5). That only requires that the eighth gluon $G_8^\mu$ is changed into $G_8^\mu/\sqrt{3}$. Then $SU(3)$ may be changed in a triple direct product of $SU(2)$, with the above geometrical implications, in the following way: one changes the eighth Gell-Mann matrix $\lambda_8$ into $\sqrt{3}\lambda_8$ and one replaces this new matrix by the sum of two suitable matrices in such a way that the eight Gell-Mann matrices are replaced by three sets of three Pauli matrices. Also such a change allows, but not imposes, to consider nine gluons instead of eight.

The replacement of $SU(3)$ by a a triple direct product of $SU(2)$ : as I am not a physicist, I can only hope that this so simple transformation has been or will be indicated elsewhere, independently or not, by some authorized physicist.

If such a transformation would be validated by the experiments, that would be an additional confirmation of the necessity of the replacement of the complex formalism by the real one.
References


Note

The above text has not been written in view of a publication.
If I have used some irony in the presentation of a nonsense inside a calculation achieved by physicists among the most eminent of the last century, nothing is changed in my admiration for their works in general and this calculation in particular.

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This book mainly focuses on the general principles and most common methods used to solve quantum mechanics problems. Don't expect it to teach you how to solve any specific problem; it won't. Instead, read this book to get a good "bird's eye view" of what it means to become involved in quantum mechanics. I'm not saying that this book is dumbed down—not at all. Every page is covered in equations. It's Quantum Mechanics, Stupid. And Now for Something Completely Different: Quantum Reality. Quantum Mechanics with the Gloves Off. Firstly, a review of a pedagogical improvement on John Bell’s famous analysis of hidden variables in quantum mechanics. It’s easier to explain than Bell’s original argument, and deserves to be widely publicized. It was built by David Mermin. He took out of some earlier work by Greenberger, Horn, and Zeilinger. In the second [main] part of the lecture I will turn to the much vexed question sometimes called the interpretation of quantum mechanics, although, as I will argue, that’s really a bad name for it. I want to stress that I have made no original contributions to this subject. Quantum mechanics is a general theory. It is presumed to apply to everything, from subatomic particles to galaxies. But interest is naturally focused on those phenomena that are most distinctive of quantum mechanics, some of which led to its discovery. The physical development of quantum mechanics begins in Ch. 2, and the mathematically sophisticated reader may turn there at once. But since not only the results, but also the concepts and logical framework of Ch. Quantum mechanics is a key known feature of physics, and also, it seems, a natural and inevitable feature of our models. In classical physics—or in a system like a cellular automaton—one basically has rules that specify a unique path of history for the evolution of a system. But our models are not set up to define any such unique path of history. Instead, the models just give possible rewrites that can be performed on hypergraphs but they do not say when or where these rewrites should be applied. In the standard formalism of quantum mechanics, one usually just imagines that all one can determine are probabilities for different histories or different outcomes. But this has made it something of a mystery why we have the impression that a definite objective reality seems to exist. When discussing physics, quantum mechanics was a recurring theme which gained prominence after his decision to write this book. He completed the manuscript three months before his death and asked me to take care of the proofreading and the Index. A labour of love. I knew what Tony wanted and what he did not want. Except for corrections, no changes have been made. Tony was an outstanding teacher who could talk with students of all abilities. He had a deep knowledge of physics and was able to explain subtle ideas in a simple and delightful style. Who else would refer to the end-point of nuclear...